MPC Controller

LINEAR MPC FOR LANE KEEPING AND OBSTACLE AVOIDANCE ON LOW CURVATURE ROADS

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Suitable System Proposal

The Model Predictive Model method aims minimize i.e., reduces to zero, the states weighted in the Cost Matrix Q. Hence, we need a model whose steady state vector is $\vec{0}$. The Overreacting lateral dynamic model, described by Linear Model Predictive Control for Lane Keeping and Obstacle Avoidance [1] is used in this work.

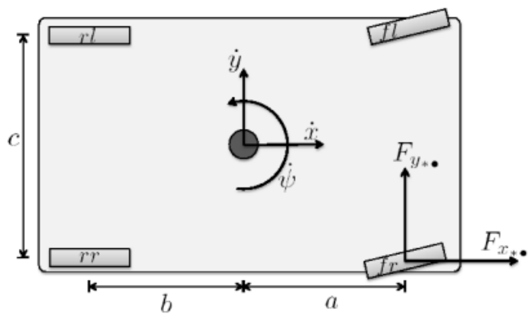


Figure 2 - Vehicle Body

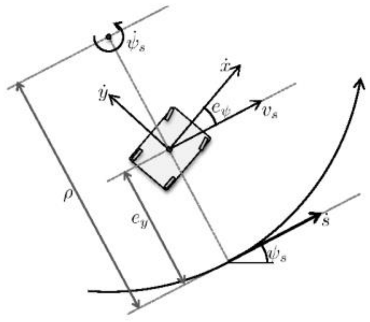


Figure 1 - Road Curvilinear Coordinate System

$$
\begin{aligned}
m\ddot{y} &= -m\dot{x}\dot{\psi} + C_{f_U}\alpha_f + C_{r_U}\alpha_r \\
I\ddot{\psi} &= aC_{f_U}\alpha_f - bC_{r_L}\alpha_r \\
\dot{e}_\psi &= \dot{\psi} - \dot{x}\psi_r \\
\dot{e}_y &= \dot{y} + \dot{x}e_\psi \\
\dot{\delta} &= u,
\end{aligned}
$$

$$
\alpha_f = \frac{\dot{y}+a\dot{\psi}}{\dot{x}} - \delta \;; \quad \alpha_r = \frac{\dot{y}-b\dot{\psi}}{\dot{x}}
$$

$$l_{width} = 3\ meter$$

$$T_{sampling} = 0.1\ sec$$

$$N_{horizon} = 30\ step$$

$$
\begin{bmatrix} \ddot{y} \\ \ddot{\psi} \\ \dot{e}_\psi \\ \dot{e}_y \\ \dot{\delta} \end{bmatrix} =
\begin{bmatrix}
\frac{C_{fu}+C_{ru}}{\dot{x}m} & -\dot{x}+\frac{C_{fu}a-C_{ru}b}{\dot{x}m} & 0 & 0 & -\frac{C_{fu}}{m} \\
\frac{C_{fu}a-C_{rL}b}{\dot{x}I} & \frac{C_{fu}a^2+C_{rL}b^2}{\dot{x}I} & 0 & 0 & -\frac{aC_{fu}}{I} \\
0 & 1 & 0 & 0 & 0 \\
1 & 0 & \dot{x} & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} \dot{y} \\ \dot{\psi} \\ e_\psi \\ e_y \\ \delta \end{bmatrix} +
\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & -\dot{x}\psi_r \\ 0 & 0 \\ 1 & 0 \end{bmatrix}
\begin{bmatrix} u_\delta \\ 1 \end{bmatrix}
$$

$$
y = e_y = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \end{bmatrix}
\begin{bmatrix} \dot{y} \\ \dot{\psi} \\ e_\psi \\ e_y \\ \delta \end{bmatrix} + \begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} u_\delta \\ 1 \end{bmatrix}
$$

Why MPC Controller

The system is linear except an DC offset in the $\dot{e}_\psi$, $-\dot{x}\psi_r$, which can be included by second constant input or excluded by state modifications. I have preferred to use a constant input which is restricted by equality constraints. The important feature is that the e_y state represents the location on the road horizontally, hence it can also be presented as the error from the centerline of the road.

All these properties of this model, allow us to benefit the MPC method, and minimizing $C'C$ matrix.

Design Formulations

Cost Matrix Selection

Cost Matrix is selected considering only the output state, e_y, such that:

$$C = [0 \quad 0 \quad 0 \quad 1 \quad 0] \rightarrow Q = C'C$$

Constraints

Model Specific Constraints

$$u_2 = [0 \quad 1]\,\bar{u} = 1 \rightarrow A\bar{u} = b$$

Car Mechanical Constraints

$$|\delta| \leq \delta_{lim} = \frac{\pi}{3}$$

$$|\dot{\delta}| = |u_\delta| \leq \dot{\delta}_{lim} = \frac{\pi}{3}$$

$$\left|\frac{\dot{y} + a\dot{\psi}}{\dot{x}}\right| < \alpha_{f,lim} = \frac{\pi}{6}$$

$$\left|\frac{\dot{y} - b\dot{\psi}}{\dot{x}}\right| < \alpha_{r,lim} = \frac{\pi}{6}$$

Model Performance Constraints

$$|e_y| \leq \frac{1}{2} l_{width} = \frac{3}{2}$$

$$e_{y_{obstacle}} < e_y < \frac{1}{2} l_{width} \quad or \quad -\frac{1}{2} l_{width} < e_y < e_{y_{obstacle}}$$

Disturbances

Road Curvature is included by approximated and linearized as $\psi_r = \frac{1}{\rho}$. But it may not be constant along the path.

The Reference Speed, $\dot{x}$, can have a random distribution around desired speed.

The “roadholding” can be randomized along the path.

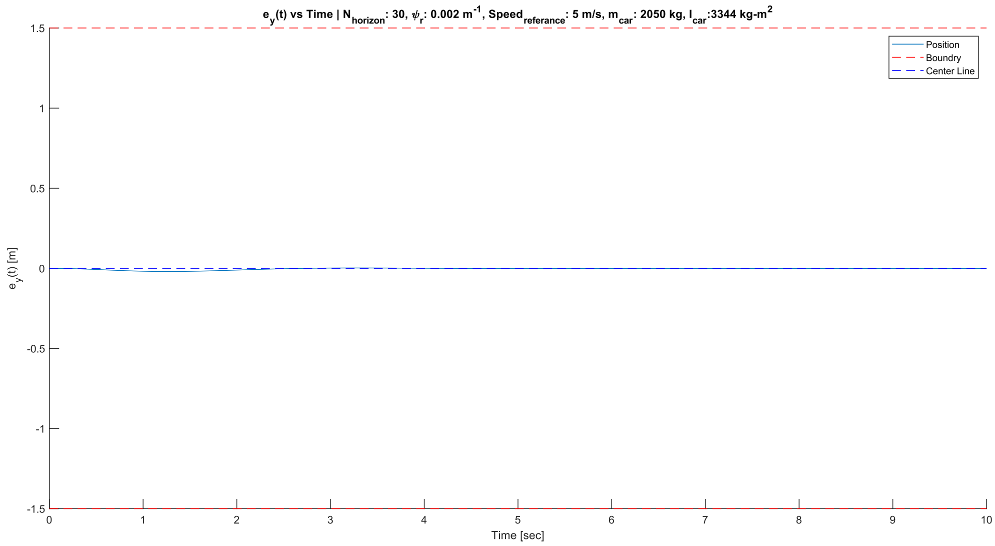
Wind effect is not considered.

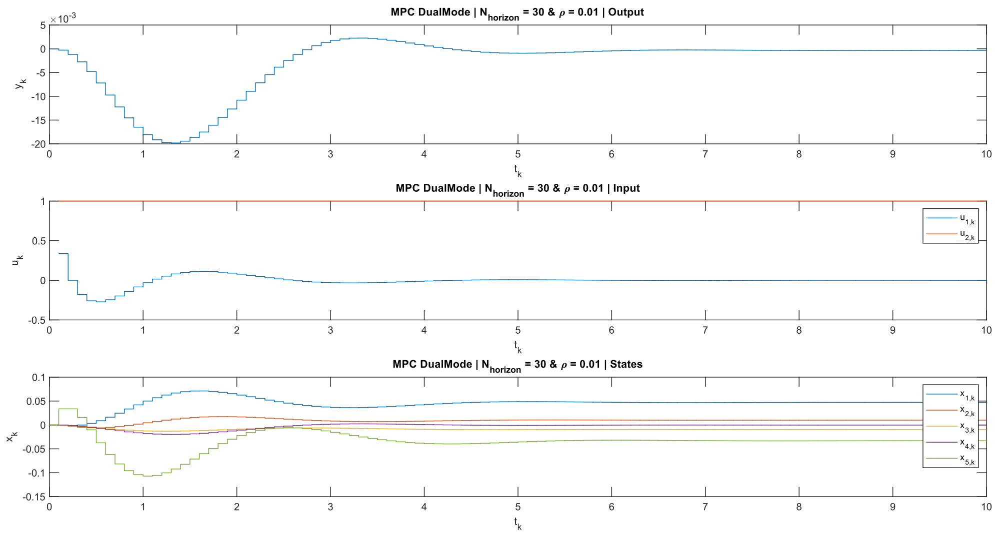
Test Results

Lane Keeping Validation

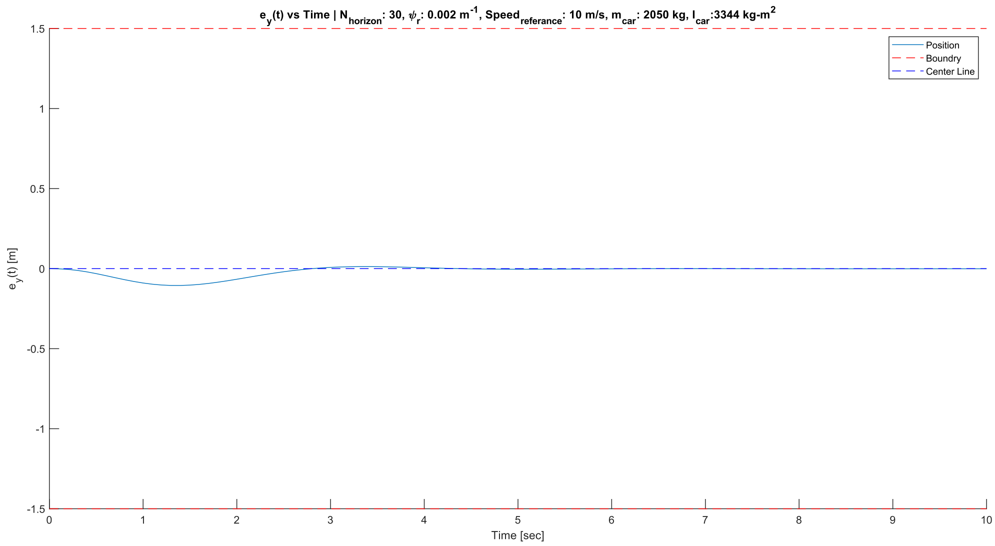
$$Let\ \rho = 500\ m \rightarrow \psi_r = \frac{1}{500} m^{-1}$$

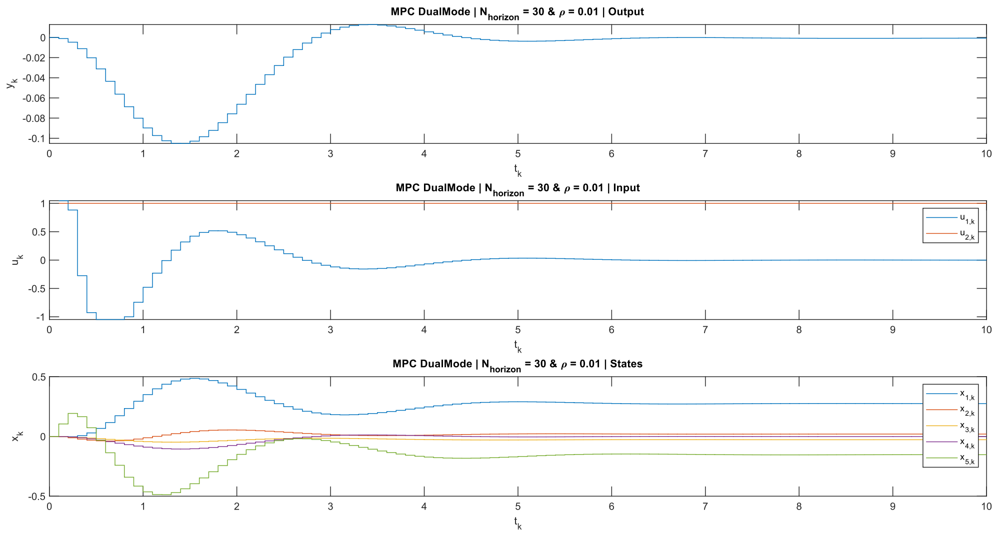
- $\dot{x} = 5\ m/s \rightarrow 18\ km/h$



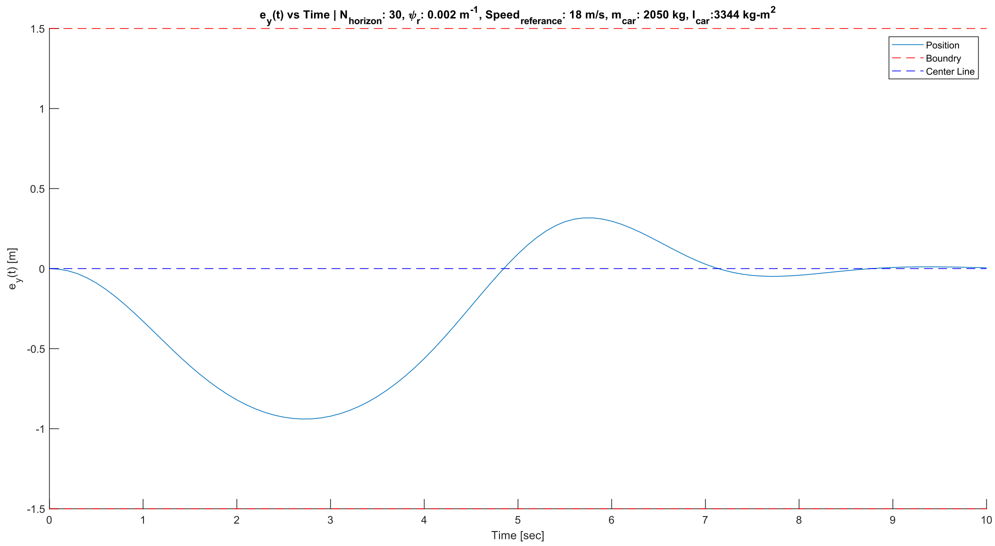


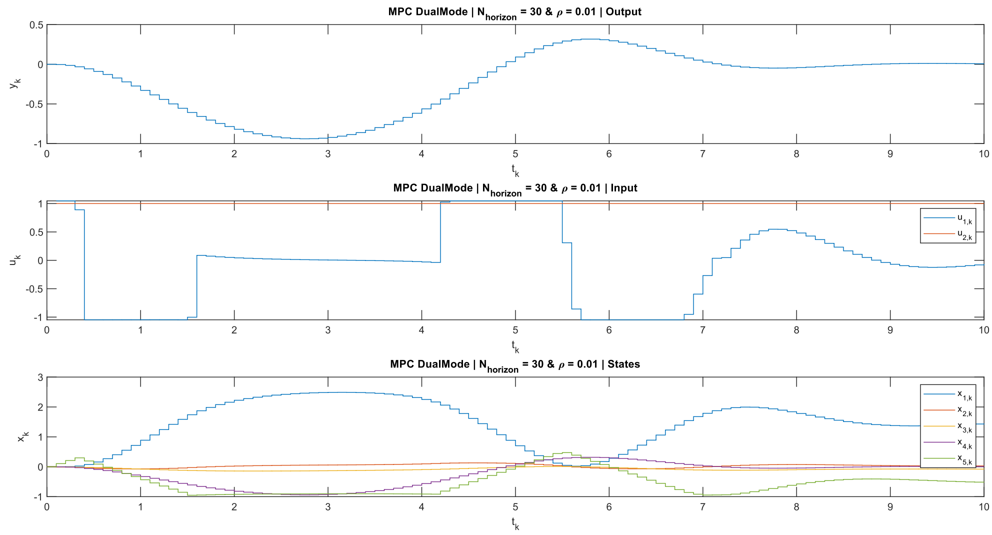
- $\dot{x} = 10\ m/s \rightarrow 36\ km/h$



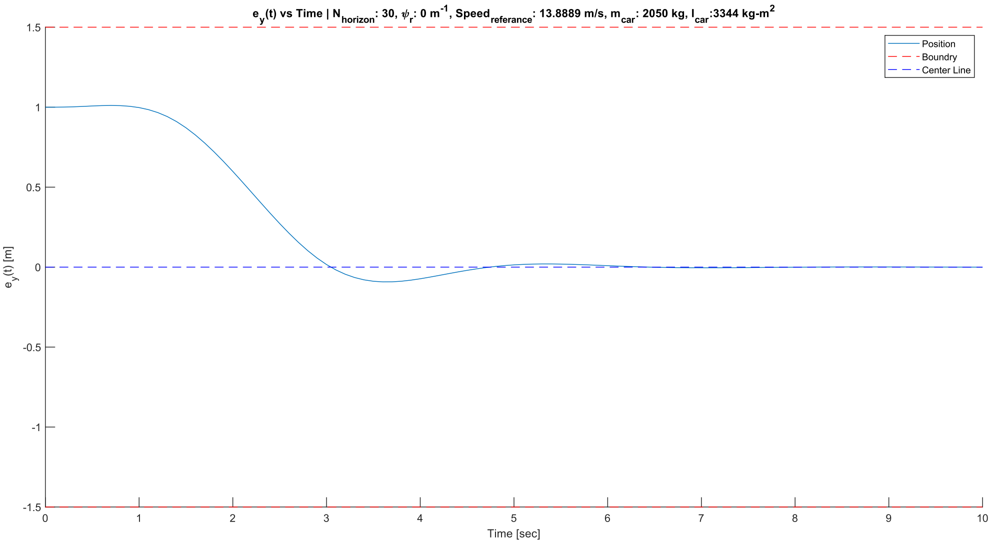


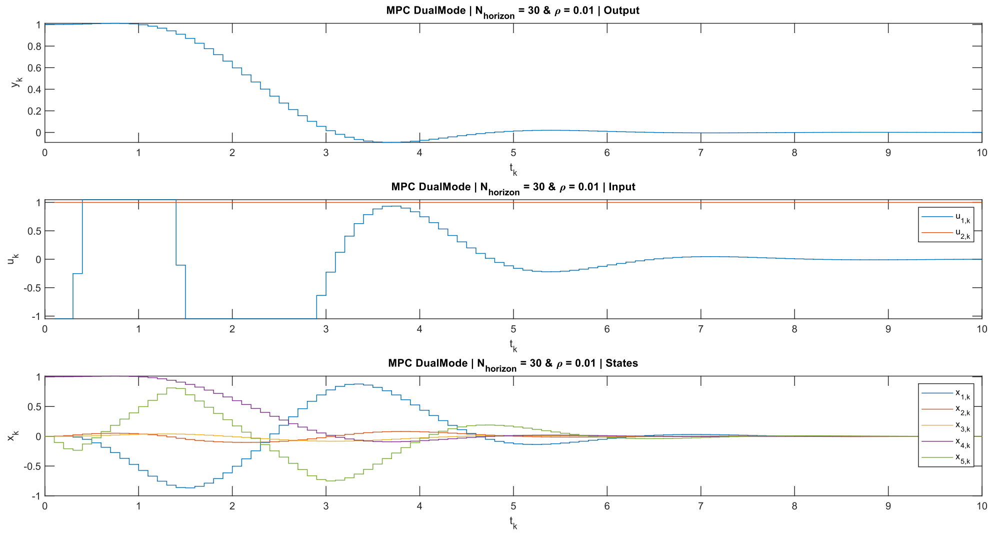
- $\dot{x} = 18\, m/s \rightarrow 65\, km/h$





- $\dot{x} = 13.889\ m/s \rightarrow 50\ km/h \mid \rho = \infty \rightarrow\ \psi_r = 0 \mid e_y(0) = 1$

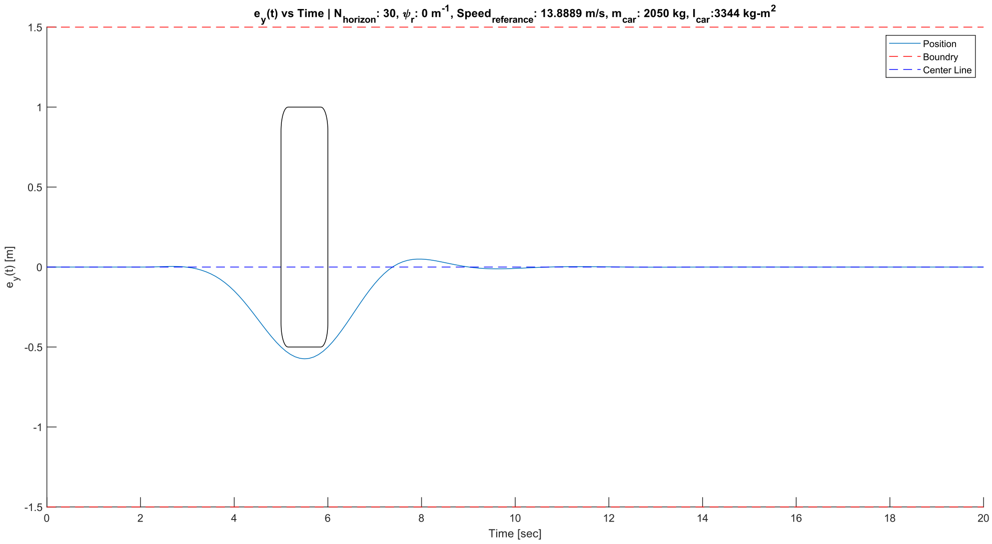


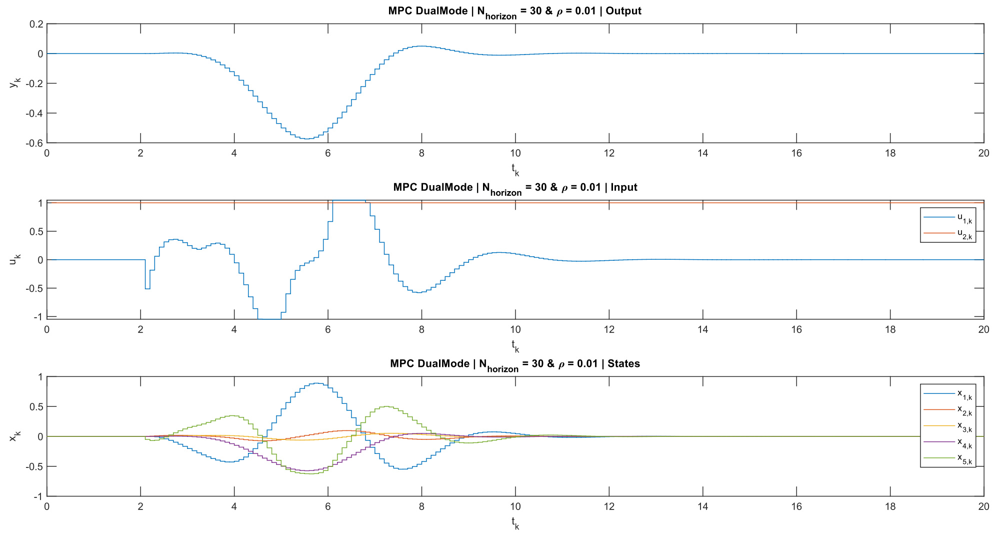


Obstacle Avoidance Validation

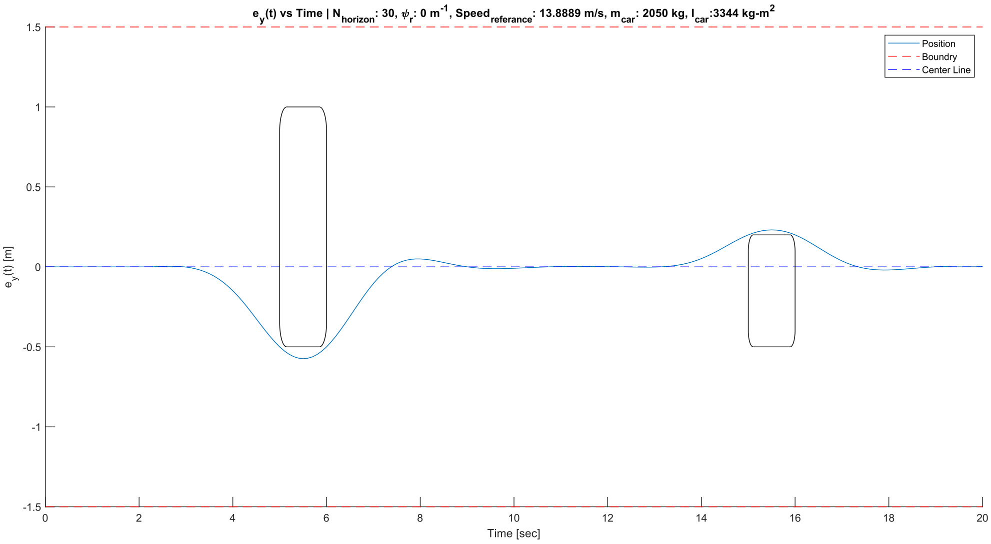
$$\rho = \infty \rightarrow \psi_r = 0 \mid \dot{x} = 50\ km/h$$

- 1 Obstacle

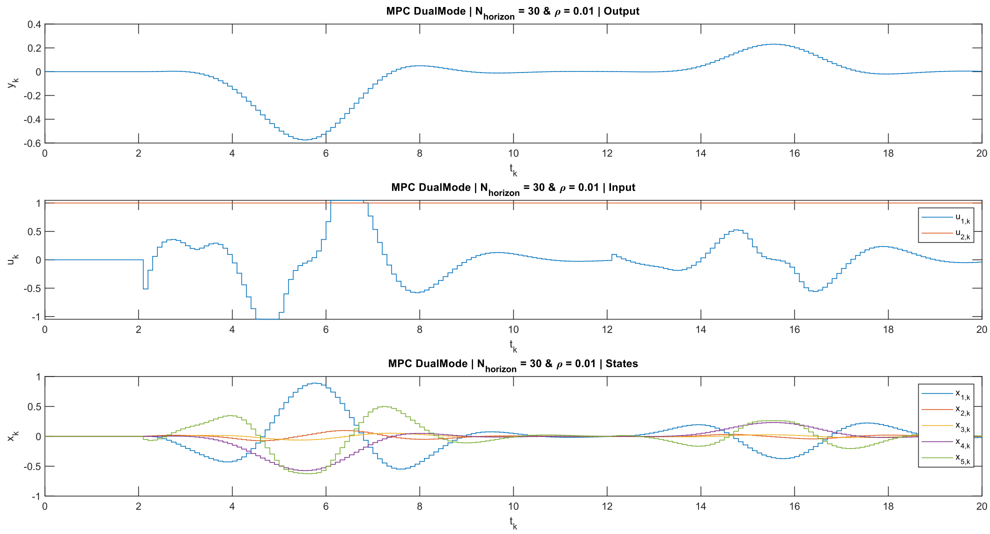




- 2 Obstacles

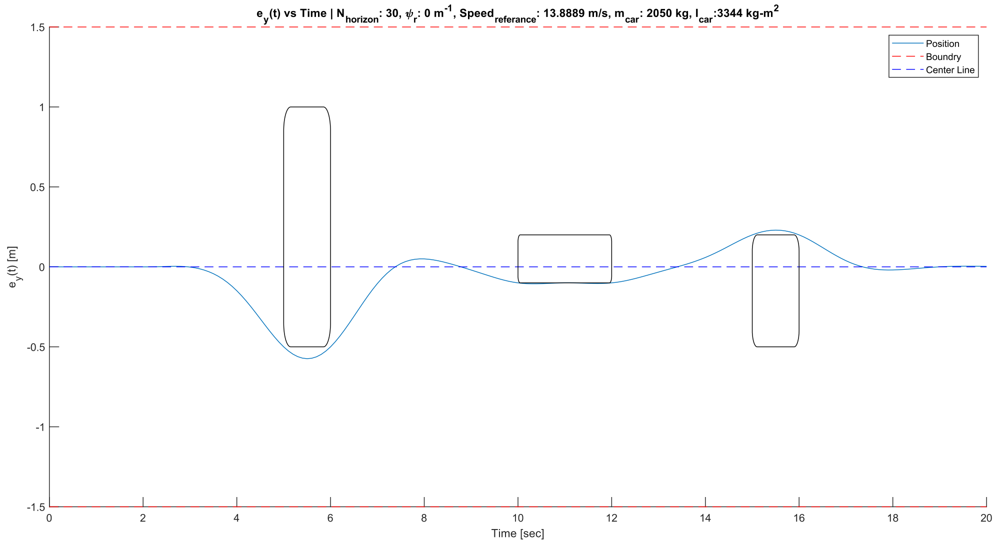


$e_y(t)$ vs Time | $N_{horizon}$: 30, ψ_r: 0 m^{-1}, $Speed_{referance}$: 13.8889 m/s, m_{car}: 2050 kg, I_{car}:3344 kg-m^2
Position
Boundry
Center Line
$e_y(t)$ [m]
Time [sec]

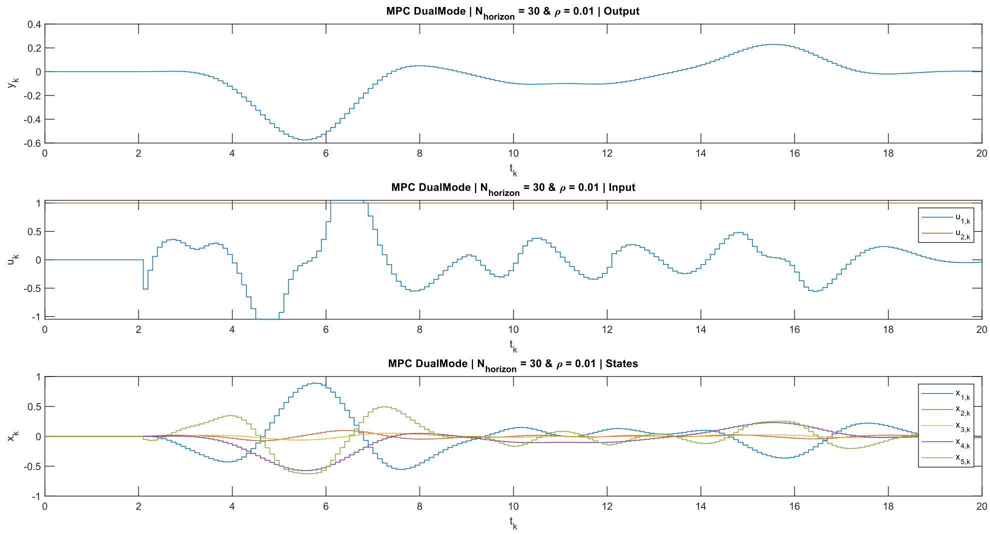


MPC DualMode | $N_{horizon}$ = 30 & ρ = 0.01 | Output
y_k
t_k
MPC DualMode | $N_{horizon}$ = 30 & ρ = 0.01 | Input
u_k
$u_{1,k}$
$u_{2,k}$
t_k
MPC DualMode | $N_{horizon}$ = 30 & ρ = 0.01 | States
x_k
$x_{1,k}$
$x_{2,k}$
$x_{3,k}$
$x_{4,k}$
$x_{5,k}$
t_k

- 3 Obstacles



e_y(t) vs Time | N_horizon: 30, ψ_r: 0 m^-1, Speed_referance: 13.8889 m/s, m_car: 2050 kg, I_car:3344 kg-m^2
Position
Boundry
Center Line
e_y(t) [m]
Time [sec]



MPC DualMode | N_horizon = 30 & ρ = 0.01 | Output
y_k
t_k
MPC DualMode | N_horizon = 30 & ρ = 0.01 | Input
u_k
u_1,k
u_2,k
t_k
MPC DualMode | N_horizon = 30 & ρ = 0.01 | States
x_k
x_1,k
x_2,k
x_3,k
x_4,k
x_5,k
t_k

References

[1] V. Turri, A. Carvalho, H. Tseng, K. Johansson and F. Borrelli, "Linear model predictive control for lane keeping and obstacle avoidance on low curvature roads," *ITSC 2013,* pp. 378-383, 2013.